

MODIFIED SPECTRA TO ACCOUNT FOR SOIL AMPLIFICATION  
A PARAMETRIC STUDY FOR THE MONTREAL REGION

by

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ABSTRACT

The dynamic response of soils is studied for a generated accelerogram compatible with the response spectrum of Commentary K of the National Building Code. Idealizing the soil medium as a one-dimensional column with non-linear shear modulus and damping, a parametric study is investigated for the Montreal region. The frequency response method is used for the analysis. The parameters that were considered are: type of soil namely sand or clay, maximum shear modulus, soil depth, base acceleration level and effect of a surface freezing on the response. The results show that the response spectrum at the surface compared to the corresponding value at the base can exceed the prescribed Code value  $F$  by a large factor. This amplification is not constant and depends obviously on the period of the soil and that of the structure.

INTRODUCTION

The object of this paper is to make structural designers aware of some of the problems that could arise when buildings resting on soils are subjected to earthquakes of variable intensity. The National Building Code of Canada (1) treats the subject of medium or soft soils by means of an amplification factor  $F$  which increases the base shear by a maximum value of 1.5. Such amplification has already been reported (2,3,4,5) and the effect of quasi-resonance between the soil and structure is not new.

On the other hand, the type of buildings that are usually considered by the National Building Code do not, in general, warrant a full soil-structure interaction analysis as in the case of nuclear power plants (6,7). The emphasis has therefore been placed here on the non-linearity of the soil properties and its damping as well on the various parameters that influence such an amplification. All the values used refer typically to the Montreal region (8). However, they can be easily used for other regions having similar properties and acceleration levels.

METHODS OF ANALYSIS

For a one-dimensional seismic amplification response the mass density, shear modulus and damping ratio are the necessary parameters required

to define the soil mass, stiffness and damping matrices respectively. If the mass density is constant and has an average value of  $2000 \text{ kg/m}^3$ , the shear modulus and damping vary with the shear distortion  $\gamma$ . Typical values are shown in Figure 1 (5,9) for sand and clay. Sand liquefaction which can be important in some cases (10) is not considered in this study. The dynamic properties of soils clearly indicate the need for non-linear analysis even if considered approximate.

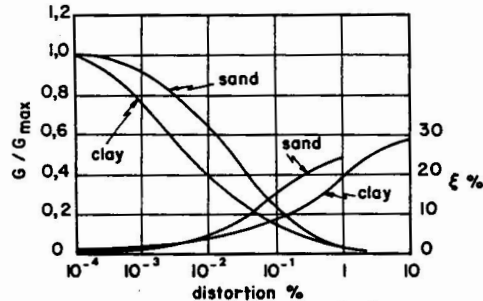


Fig. 1 Typical variation of shear modulus and damping with shear distortions.

Three approaches are usually considered (2) for the evaluation of the soil amplification:

1. Select a single value for the shear modulus and damping ratio and perform a linear analysis in the time or frequency domain.
2. Use the results of a linear analysis and select new values of modulus and damping consistent with these results and iterating until values from two sequential cycles are within specified limits. Each cycle is treated therefore as a linear analysis that can be performed in the frequency domain or by using the modal technique.
3. Perform a two or three-dimensional dynamic analysis with Finite Elements and solve the system of equation through direct integration in the time domain. At each time increment the non-linear properties are adjusted based on the results of the previous increment.

With a one-dimensional analysis, the second approach is the one adopted here for both the modal technique and the frequency response method. The equations of motion are usually written in the form:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = -[M]\{I\} \ddot{a}_g \quad (1)$$

where  $\{U\}$  is the relative displacement and  $\ddot{a}_g$  is the base acceleration. The matrices  $[K]$ ,  $[C]$  and  $[M]$  refer to the assembled stiffness, damping and mass matrices respectively. The  $i$  element stiffness and mass matrices are simply given by

$$[k]_i = \frac{G_i}{h_i} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad [m]_i = \frac{\rho h_i}{2} \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \quad (2)$$

where  $G_i$  is the shear modulus,  $h_i$  the corresponding element height and  $\rho$  the mass density of the soil.

In the modal technique, the damping matrix [C] is frequency dependent and is usually a function of either [K] and/or [M] in order to satisfy the orthogonality conditions. Furthermore, the damping cannot to be varied easily within an element or within a layer; its value being constant for a given mode.

In the frequency domain (2), the equations of motion for a given value of  $\omega$ , the natural circular frequency, are rewritten as follows:

$$([K]^* - \omega^2[M])\{U(\omega)\} = -[M]\{I\} \ddot{Y}_g(\omega) \quad (3)$$

where the ground acceleration is decomposed in a Fourier series  $\ddot{Y}_g(\omega)$  using the FFT (11). Equation (3) is in complex form with

$$[K]^* = [K] + i\omega[C] \quad (4)$$

The damping ratio should be independent of the frequency when hysteretic damping characterizes soil behaviour. This is obtained (2) by treating the shear modulus for each element as:

$$G^* = G(1 - 2\xi^2 + 2i\xi\sqrt{1 - \xi^2}) \quad (5)$$

where  $\xi$  is the damping value corresponding to Figure 1.

The accelerogram used in this study has been generated by modifying the El Centro earthquake through a filter and then using the suppressing and raising technique (12). The new accelerogram obtained is shown in Figure 2 and its corresponding spectrum in Figure 3 which is very close to the spectrum of Commentary K (13). The new spectrum SIMUL is normalized to 1 g and has a duration of 30 seconds.

Two programs using the modal technique (MODALDS) and the complex frequency response (FREQDS) have been developed (14). Either method can be used once an accelerogram has been chosen. In order to test the validity and accuracy of the programs, an extremely arbitrary stiff soil has been analysed. The spectrum obtained at the surface was identical to the one at the base. Finally, in order to evaluate the effect of different accelerograms on the soil response, three accelerograms have been selected. Figure 4 shows the soil amplification using the accelerograms for ELCENTRO, TAFT and SIMUL. The maximum acceleration was set at 4% and the soil parameters used are shown on the figure. The spectra at the surface correspond to a damping ratio  $\xi = 2\%$ . This value is used throughout. The correlation is quite close in this case. Other cases have been simulated and are reported in (14).

A point of capital importance relates to the number of masses required to idealize a soil layer correctly (2). Knowing the modulus G, the mass density  $\rho$  and the depth H, the number of masses  $n$  is given by

$$n > \frac{8H}{T_{\min}\sqrt{G/\rho}} \quad (6)$$

where  $T_{\min}$  is the minimum expected period.

#### PARAMETRIC STUDY

For one-dimensional columns of soil, a number of parameters can be studied. These are: the non-linear variation of shear modulus and

damping with distortion, the maximum shear modulus  $G_{max}$ , the soil depth  $H$ , the water table level, the accelerogram used as well as the acceleration level at the base and the mass density.

For the Montreal region the parametric study, using the frequency response method and the generated accelerogram SIMUL, was limited to sand, clay or to a two-layer combination of both. Typical heights for such deposits vary between 5 and 30 m. Although Montreal is considered seismically in Zone 2, with an acceleration level of 4% g, the values of 8 and 12% g were also considered. The water table level was always set at the surface and the average mass was fixed at 2 000 kg/m<sup>3</sup>.

The values of  $G_{max}$  are defined (15) by the relation:

$$G_{max} = 3260(OCR)^{K_p} \frac{(2,973 - e)^2}{(1 + e)} \sqrt{\sigma'} \quad (7)$$

where OCR is the overconsolidation factor,  $K_p$  is a factor related to the plasticity index for clay,  $e$  is the void ratio and  $\sigma'$  the effective vertical stress.

For the case of sand,  $K_p = 0$  and the values of  $e$  were chosen such that  $G_{max1} = 10\,000\sqrt{\sigma'}$  KPa and  $G_{max2} = 20\,000\sqrt{\sigma'}$  KPa. These values are considered as typical bounds for sand. Figure 5 shows for different sand depths, the Spectrum Amplification Ratio (SAR) which is the ratio of the spectrum at the surface over the spectrum at the base. The results show that the maximum values of SAR with peaks reaching a value of approximately 7 are particularly noticeable for low period buildings having a value less than 1 second. For buildings with period higher than 1,5 second, the value of SAR  $\approx 1$ . For each value of  $G_{max}$  various levels of acceleration are within the bands shown in Figure 5.

From the possible range of values of  $G_{max}$  for clay, the values chosen here were such that the different dynamic responses thus obtained would be enhanced. This resulted in values for  $G_{max3} = 2\,500\sqrt{\sigma'}$  KPa and  $G_{max4} = 5\,000\sqrt{\sigma'}$  KPa. The values of the SAR are shown in Figures 6 and 7 for the same soil depth and acceleration levels. The results show a larger range of periods affected by the SAR. In general, these values are slightly lower than the ones in Figure 5. However, the effect of the acceleration level, as a parameter, tends to increase the band for a given rigidity, compared to the results for sand.

The results of Figures 5, 6 and 7 summarize the influence of depth as a parameter. If the acceleration level is now considered, Figures 8 and 9 show the SAR for 4, 8 and 12% g. These figures represent the envelopes of previous values. For both sand and clay, the SAR is generally larger for smaller acceleration levels. However, the values of SAR for sand are well confined to structures with periods below 1,5 second which is definitely not the case for clay.

#### SPECIAL CONDITIONS

The first case refers to the influence of having a bi-layered medium composed of both sand and clay. The case studied is 30 m deep subjected to an acceleration level of 4% g. Figure 10 shows the results of SAR. The results for the case where clay is below the sand is fairly close to

the medium with clay alone. The other curves where sand is below the clay a different behaviour is noticed compared to the case of sand alone. For this latter case, if  $G_{max}$  is set at  $20\ 000\sqrt{G^T}$  KPa, the difference is even more pronounced.

The second case of special interest refers to a two-layered medium where the top part is kept constant at 1,5 m in depth and is considered frozen. This rigidity was assumed to be 1 000 times larger than the conventional value and the damping ratio was set at 1%. The lower layer is either sand or clay and the relevant properties are shown in Figure 11. The results refer to the ratio of the spectrum at the surface (considering the frozen layer) over the spectrum without the frozen part. For an acceleration of 4% g, the results are always less than unity. However, the effect of a frozen surface layer can influence the liquefaction of sand which, in this study, has not been considered. Indeed the presence of a frozen surface layer acting as an impervious zone will increase the danger of liquefaction of the saturated sand present below.

#### CONCLUSIONS

- 1) The F factor considered by the NBCC is dependent on the depth, rigidity, presence of multi-layers and acceleration level of the earthquake. This value can reach in case of quasi-resonance, in sand, a value of approximately 7. For buildings with periods greater than about 1,5 second, the F factor is close to unity. As for clay, the amplification factor is smaller but covers a wider range of periods since clay is more sensitive to the parameters defining it.
- 2) The amplification factors are generally higher for lower acceleration levels. This is primarily due to the non-linear behaviour of the damping.
- 3) Results of multi-layered soils clearly indicate the difficulty in predicting accurately the SAR using empirical formulations. Also the effect of a frozen layer near the surface has little influence on the amplification when liquefaction is not possible.
- 4) The program developed in this study are very easily used in an interactive fashion with all the results reproduced graphically.
- 5) For more complex problems where uni-dimensional column elements are not sufficiently accurate, a two or three-dimensional finite element model is the next logical step in order of complexity.
- 6) Finally, the parametric study performed here indicate the influence of soil properties on the response. Considering the large difference these can have on the amplification, a probabilistic approach should be considered not only for the acceleration (16) but also for the spatial variability and estimation errors of the soil properties.

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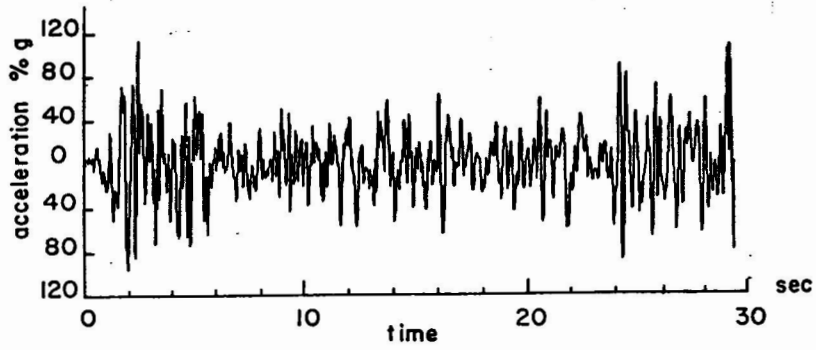


Fig. 2 Generated Accelerogram SIMUL.

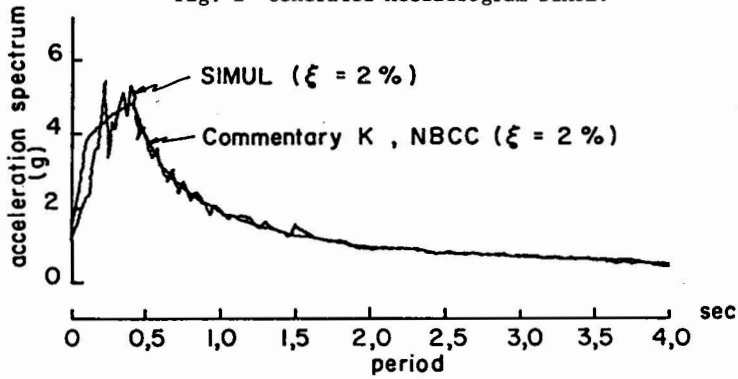


Fig. 3 Comparison of SIMUL spectrum with NBCC.

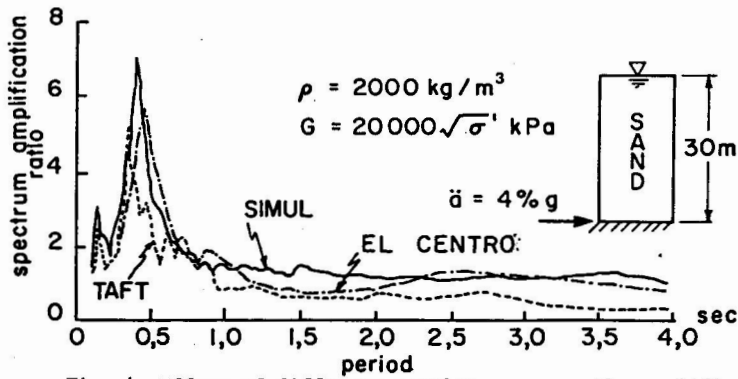


Fig. 4 Effect of different accelerograms on the amplification of the NBCC spectrum ( $\xi = 2\%$ ).

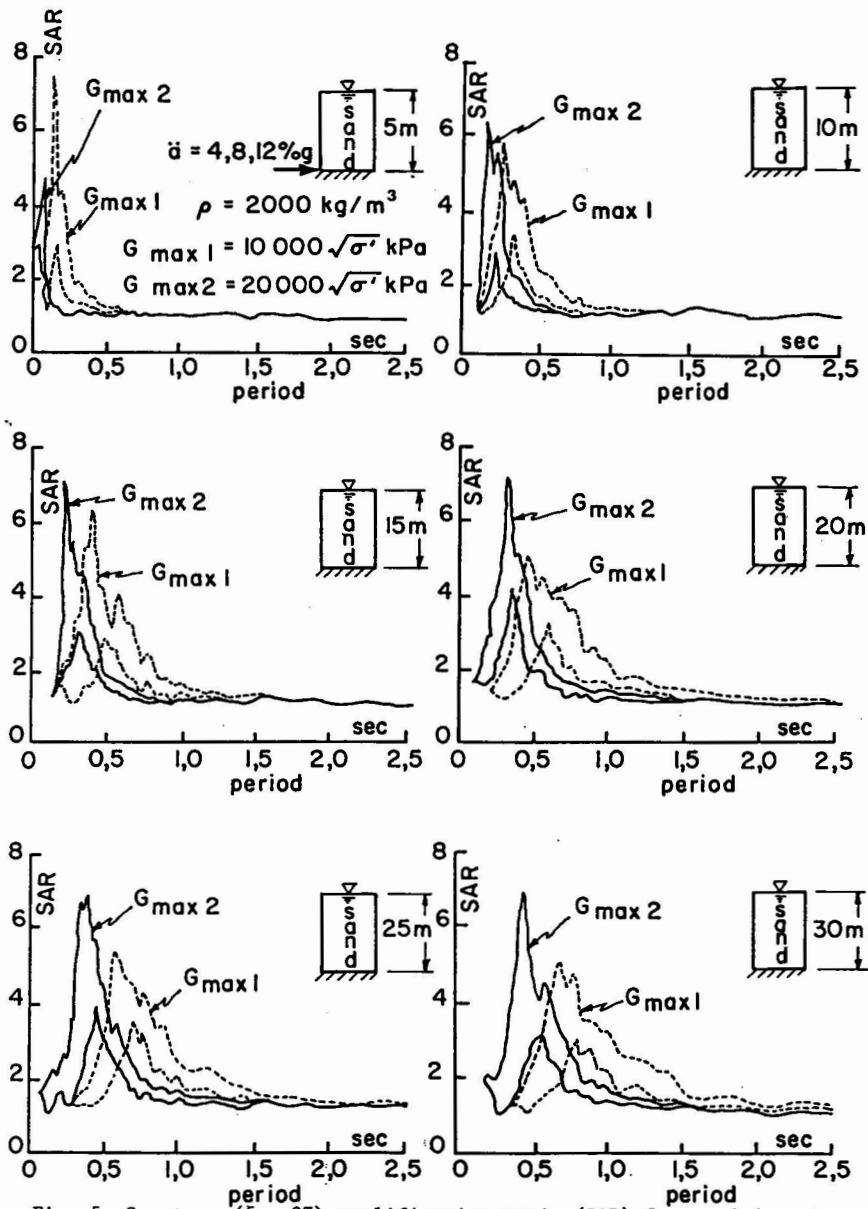


Fig. 5 Spectrum ( $\xi = 2\%$ ) amplification ratio (SAR) for sand deposits with different depth, rigidity and acceleration levels.



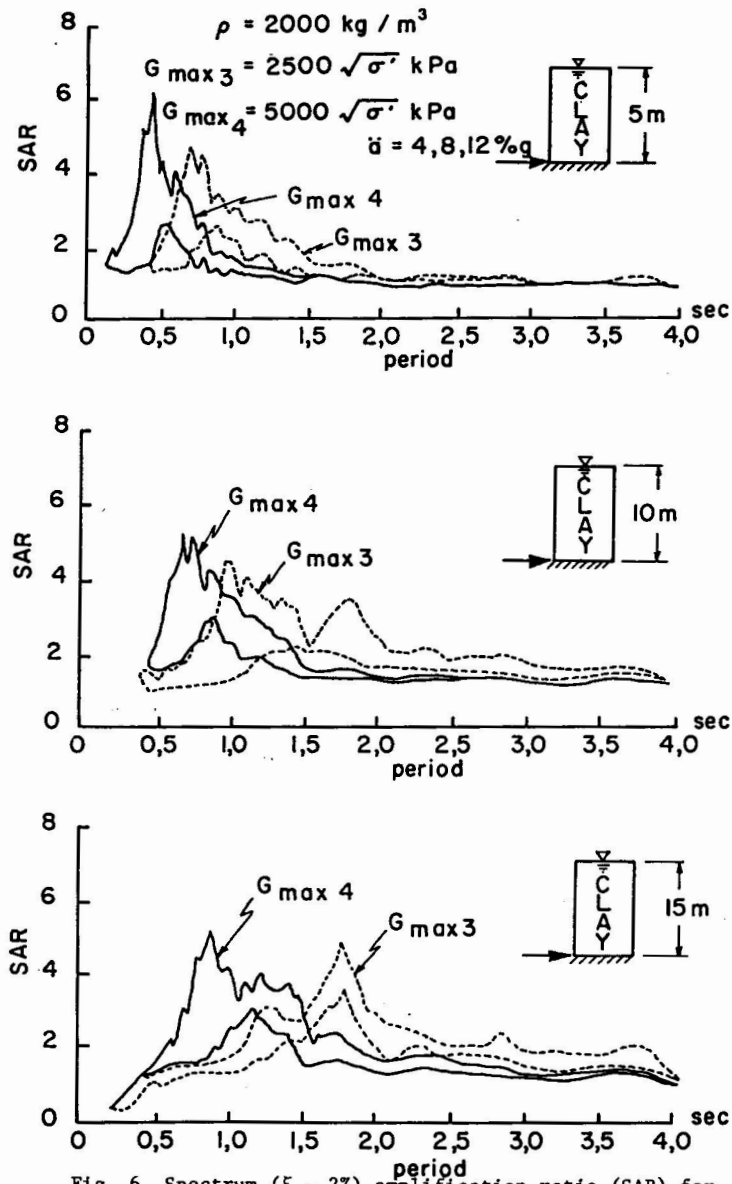


Fig. 6 Spectrum ( $\xi = 2\%$ ) amplification ratio (SAR) for clay deposits with different depth (5-15 m), rigidity and acceleration.

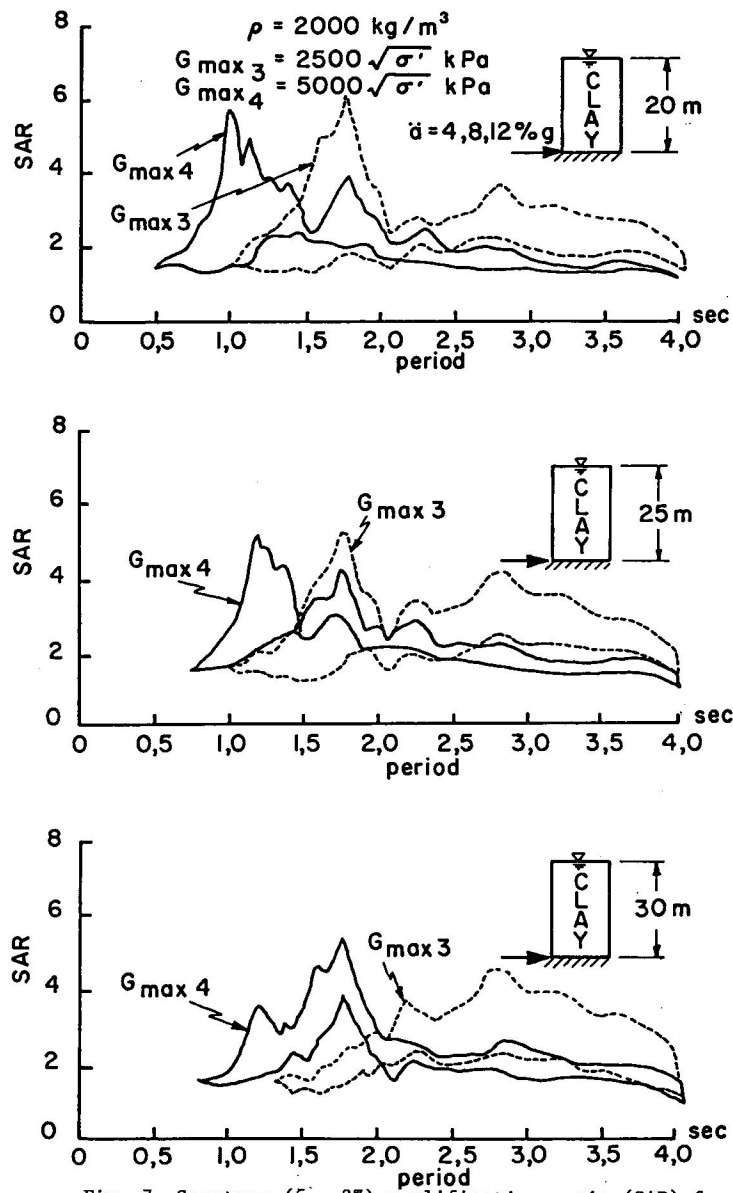


Fig. 7 Spectrum ( $\xi = 2\%$ ) amplification ratio (SAR) for clay deposits with different depth (20-30 m), rigidity and acceleration levels.

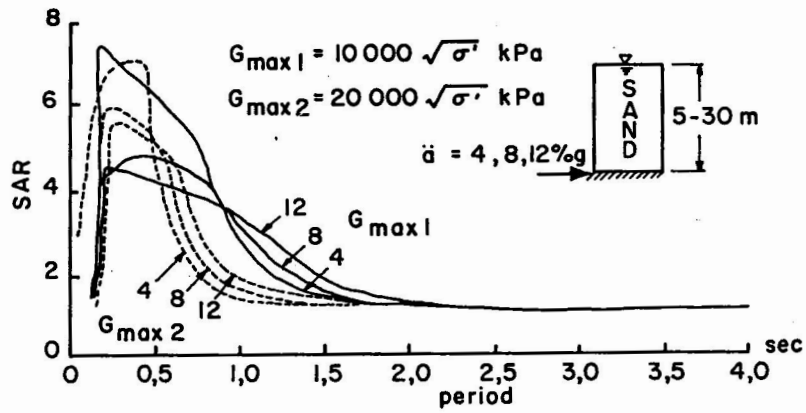


Fig. 8 Influence of acceleration levels on the spectrum ( $\xi = 2\%$ ) amplification ratio (SAR) for sand.

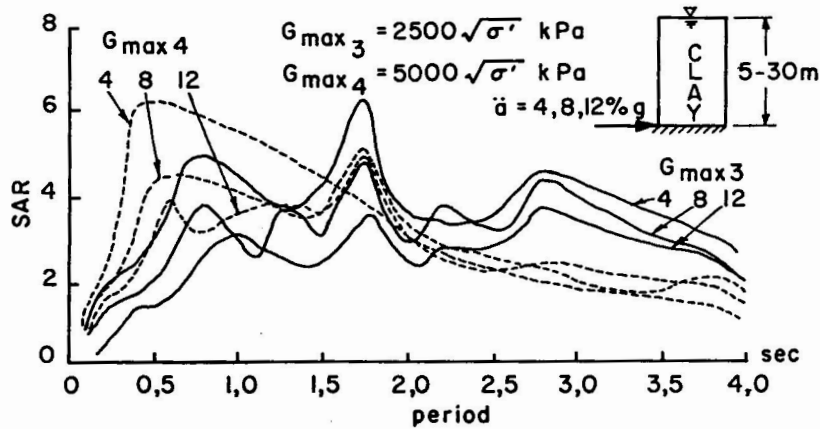


Fig. 9 Influence of acceleration levels on the spectrum ( $\xi = 2\%$ ) amplification ratio (SAR) for clay.

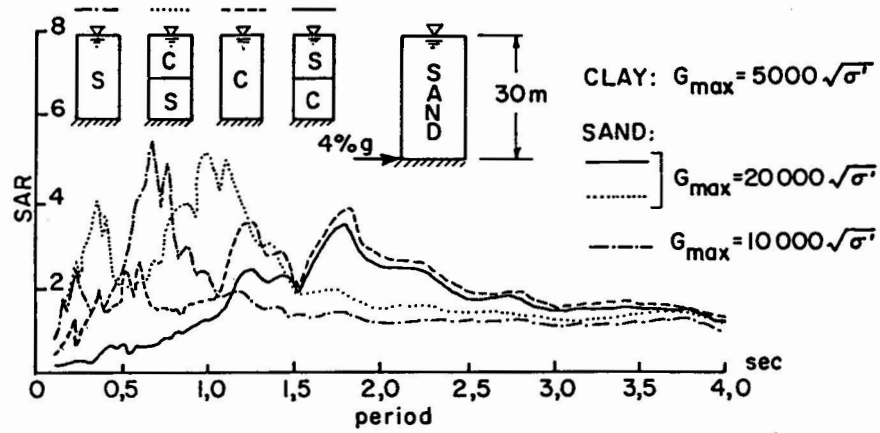


Fig. 10 Influence of multi-layers on the spectrum ( $\xi = 2\%$ ) amplification ratio (SAR).

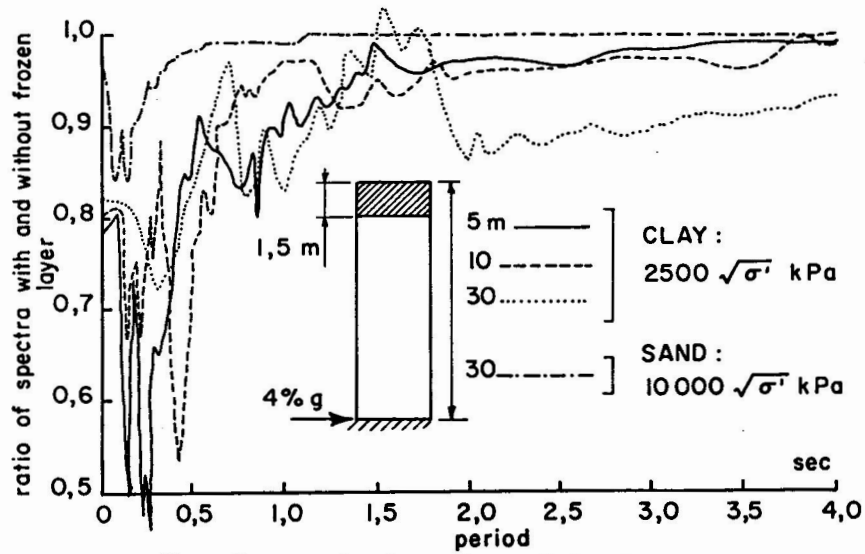


Fig. 11 Influence of a frozen layer (1.5 m deep) on the spectrum ( $\xi = 2\%$ ) amplification ratio (SAR).